Teaching within the Zone of Proximal Development

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One of the very few concepts that appears in more popular notions about learning and development is the zone of proximal development. Vygotski described the zone of proximal development as the difference between a learner’s current level of performance and what the learner can learn with assistance. In other words, the learner’s current knowledge implies some of things he can learn immediately with proper instruction.

Below is a graphic representation of the variables as they are currently configured.¹

Vygotsky’s Zone of Proximal Development

The axes are labeled *level of challenge* and *level of competence*. If the arrow has the proper angle, new learning-performance will occur (without excessive anxiety or boredom). The horizontal arrows show what level the learner is able to achieve currently, without assistance, and what the learner will be able to achieve after receiving some form of scaffolding, which means support or teaching by someone who knows the skill or operation the learner is to be taught. The process is repeated with the two horizontal arrows progressively moving up the ramp, which signifies that the learner is learning new component skills.

This scheme is referred to as “focused teaching.” A better term about how it is currently interpreted would be “focused mother henning.” The scheme doesn’t have an analytical foundation that describes what is proximal, no tests of whether a concept or operation is “proximal” for particular learners, and no empirical evidence that serves as a knowledge foundation to predict the results of interventions that are proximal or not proximal for a particular learner. More problematic, the scheme is designed for individual learners, not for a group of learners who perform at the same level with respect to acquiring particular concepts. This group context occurs in schools, whenever the teacher teaches new material—information or operations.

This lack of rigor is unfortunate because the notion of sequencing material that is to be taught within the learners’ zone of proximal development applies to any complex skills that are to be taught. In other words, everything that is to be taught in schools should be within the zone of proximal development of all learners who are properly placed.

Consider the alternative: The material presented is not within some learners’ zone of proximal development. Those learners fail to learn the material because it contains elements they have not learned and must somehow be intuited. In the meantime these learners are not able to perform as expected by the teacher. They are guessing at words and stumbling over math operations.
No sensible analysis of instruction would sanction a diet of this type of failure. As the diagram above shows, if the angle of the arrow is too steep, the instruction induces anxiety, which is something that occurs with frightening regularity at all grade levels and all subjects. The instruction also results in failure, not because the learners couldn’t learn the material if it were properly sequenced but simply because parts of what was taught were not proximal to what the learners knew. The solution to this problem is to design the sequence so that everything taught is within the learners’ zone of proximal development.

**Rating Interventions**

An effective instructional scheme would reference the learner’s current skill level in clear language. The scheme would clearly express the targeted change in the behavior of learners who go through the teaching sequence and indicate exactly how the new content will be taught. The scheme would address the length of time that will be required to teach the targeted content, and details of how the new content is to be taught.

If the planned instruction succeeds, we would know that the design is within the learners’ zone of proximal development, but we wouldn’t know relatively how successful it is. With the data we have on the targeted instruction, we could compare our approach with other options for teaching the same component skills. If another approach taught a higher percentage of learners in less time, it would be judged superior to the approach that we originally used.

**The Targeted Knowledge Gap**

A difficult question is: How wide can gaps be between what the learners currently know and what their performance will be following the teaching? The answer seems to be at least partly empirical. Ideally, sequences would be tested with teachers and students before the specific practices are disseminated to other teachers. The size of the gaps would be based initially...
on the designers’ best guess. The results would clearly disclose which teachings were in the learners’ zone of proximal learning and which were not. The implied remedy for learners’ failure would be to reexamine the gap and try to identify the missing instruction needed to bridge the gap.

A basic axiom is that anything learners need to know to learn but do not know should be taught before the learners are required to perform on tasks that incorporate that element. The teaching that is required to fill that gap may be a single element (for a proximal gap) or a sequence of elements (for a gap that is less proximal).

**Example of a Single Element the Learners Don’t Know**

Among the simplest gaps to identify are words learners don’t understand. For instance, a word that is coming up in an exercise designed for second graders requires them to understand the meaning of *lethal*. The teaching is in the learner’s zone of proximal teaching if the learner understands the definition the instruction provides. Here’s a definition that is probably sufficient.

**Lethal:** “Something that is lethal could kill you.”

The teaching could take a minute over two or three days that precede learners’ exposure to the word in the instruction that has a gap.

**DAY ONE**

(Display: *Lethal*)

This word is lethal. What word? (Signal.) *Lethal.*

Spell lethal. (Signal.) *L-E-T-H-A-L.*

Something that could kill you is lethal. A gun could be lethal because it could kill you. What are some other things that could be lethal? (Call on students. Ideas: poison, disease, a tiger, the weather, etc.)

What word tells about things that could kill you? (Signal.) *Lethal.*

**DAY TWO**

Last time, you learned a word for something that could kill things.
What word? (Signal.) *Lethal.*
Yes, something that could kill you is lethal.
What word? (Signal.) *Lethal.*
Spell lethal. (Signal.) *L-E-T-H-A-L.*
Who can make up a sentence that uses the word lethal? (Call on a few students. Accept reasonable responses. Direct the class to repeat good sentences.)

The gap has been filled and the targeted activity is now proximal for these students.

**Example of a Complex of Elements Learners Don’t Know**

An operation to be taught may seem to be a simple extension of what learners know but the gap may require teaching an elaborate sequence of elements.

Example: Learners know the alphabet. The teacher wants to teach learners how to arrange words in alphabetical order. The gap between what they know and what they would be expected to learn is not a single skill but a series of possibly six skills arranged so there is a small gap between them:

1. arranging single letters in alphabetical order;
2. arranging words in alphabetical order according to their first letter;
3. arranging words in alphabetical order according to their second letter;
4. arranging words in alphabetical order according to their third letter;
5. arranging words in alphabetical order according to their fourth letter;
6. arranging words in alphabetical order according to their first, second, third, or fourth letter.

Note that the basic wording the teacher presents is the same for all words.

1. **Ordering Single Letters.** This is the first skill needed to fill the gap.

(Display: **p k e b n z o d**)
You’re going to put these letters in alphabetical order.
Raise your hand when you know which letter comes before any of the others in the alphabet. √
Which letter comes before any of the others? (Signal.) B.
Raise your hand when you know which of these letters comes right after B. √
Which letter? (Signal.) D.
Which letter comes after D? (Signal.) E.
Which letter comes after E? (Signal.) K.

2. Ordering Words According to First Letters. After students have learned to arrange different groups of letters in alphabetical order, they would work on word sets ordered according to the first letters.

(Display: pig dig wig big rig fig jig)
You’re going to put these words in alphabetical order. The first letter of each word is underlined. The letters are p, d, w, b, r, f, j.
Raise your hand when you know which underlined letter comes before any other in the alphabet. √
Which letter? (Signal.) D.
So which word comes first? (Signal.) Dig.
Etc.

Note that this exercise incorporates the initial discrimination taught: Which letter comes first, next, etc. Learners would work with other sets that did not have the first letter underlined.

3. Ordering Words According to Second Letters. After working with several sets of words, learners would work on words that had the same first letter, but different second letters.

(Display: lot lap lunch lend lip)
You’re going to put these words in alphabetical order.
All these words start with the same letter. So you have to use the second letter to put the words in alphabetical order.

The second letters are o, a, u, e, i.

Which of those letters comes first in the alphabet? (Signal.) A.

So which word comes first? (Signal.) Lap.

Which underlined letter comes next in the alphabet? (Signal.) E.

So which word comes next? (Signal.) Lend.

Etc.

After working with several sets of words that are ordered on the basis of the second letter, learners would work with word sets that have words ordered by either the first letter or the second letter.

4. Ordering Words According to Third Letters. Next, learners would work with sets that have the same first two letters: lots, lone, love, loaf, lord. Initially, these words would not be underlined. With the earlier practice they’ve had, learners should know how to search for the third letter.

5. Ordering Words According to Fourth Letters. Next learners would work with sets that have the same first three letters: let, letting, lethal, lets, letdown. Learners would be taught that let comes first in alphabetical order because a word with no fourth letter comes before all words that have a fourth letter.

6. Ordering Words According to First Letters, First 2 Letters, First 3 Letters. This sequence is possibly over-specified, which means that a sequence that didn’t provide all the forms of practice suggested above would be effective. The extra practice, however, would not harm the learners, but would simply make the instruction easier.

If the teacher did nothing more than provide a quick explanation and then presented tasks that required learners to arrange words in alphabetical order, many learners would fail and have a high level of anxiety.
This orientation to creating small steps demands a design and data. Teachers are poor at making up tasks and activities as they teach. They need a plan that specifies what to do, what to say, how to present the examples, and how to correct learner mistakes.

**Operations That Have Concealed Steps**

When things are taught in a sequence that keeps all new learning within the zone of proximal teaching, the sequence often includes teaching that rarely occurs in traditional instruction. An example is teaching learners to change a fraction into 1 by multiplying the fraction by something. This skill is usually embedded in a series of steps learners learn to take when they learn to multiply a value by its reciprocal. This teaching is technically poor because the gap involves several steps that are not typically taught separately, but are amalgamated in an operation that is not within many learners’ zone of proximal learning. So the failure rate is fairly high.

If we pre-teach the skill of multiplying to turn a fraction into 1, we can streamline the instruction so learners learn the operation faster and more reliably.

The skill of multiplying to change a value into 1 is within the learners’ zone of proximal learning if: a) learners understand that letters represent unknowns or numbers (K = ?, B = 9) and b) if learners are able to multiply fractions such as

\[
\frac{3}{4} \times \frac{7}{5} = \frac{21}{20}
\]

and identify or create fractions that are more than 1, equal 1, and less than 1.

Here’s a problem that requires the whole operation:

\[
\frac{3}{5} M = 30
\]

Students are to figure out what 1M equals. The strategy they use is to change 3/5 into 1. The procedure is to multiply 3/5 by its reciprocal, 5/3. The equation
is changed in an unacceptable way unless the other side of the equation is also multiplied by 5/3. The result is:

\[ 1M = 30 \times \frac{5}{3} \]

So \( 1M = 50 \).

The first exercise in the gap would have learners identify whether fractions equal one. Examples:

\[
\frac{7}{9} \quad \frac{8}{8} \quad \frac{9}{5} \quad 3 \times \frac{4}{12} \quad \frac{15}{7} \times 2 \quad \frac{3M}{M} \quad \frac{4K}{K} \quad \frac{2-3}{3-2}
\]

Next, learners would learn the procedure for changing any fraction into 1. The rule they follow is: Turn the fraction upside down and multiply by that fraction. Example:

(Display: \( \frac{4}{3} \times \frac{1}{1} = 1 \))

What’s the fraction? (Signal.) \( \frac{3}{4} \).

What’s \( \frac{4}{3} \) turned upside down? (Signal.) \( \frac{3}{4} \).

Multiply \( \frac{4}{3} \) times \( \frac{3}{4} \) and see if you get a fraction that equals one. √

Say the problem for the top numbers. (Signal.) \( 4 \) times \( 3 \).

What does it equal? (Signal.) \( 12 \).

Say the problem for the bottom numbers. (Signal.) \( 3 \) times \( 4 \).

What does it equal? (Signal.) \( 12 \).

What does \( \frac{12}{12} \) equal? (Signal.) \( 1 \).

If learners work 4 problems a day for 3 days, they would understand the procedure and be able to apply it to multiplying by the reciprocal to solve problems like \( \frac{2}{3}V = 12 \). To solve for \( 1V \), learners multiply \( \frac{2}{3} \) by \( \frac{3}{2} \) and multiply the other side of the equation by \( \frac{3}{2} \).
Notice that the language specified for this sequence does no refer to numerator and denominator. The reason is that these names are inert baggage for basic math. If learners are taught both words at the same time, some learners will confuse them. If the teacher then refers to numerator and denominator in explanations, these learners may be greatly confused. The confusion can be avoided by not using obscure words and teaching them later when possible. The confusion would then have a less deleterious effect.

**Teaching Math to Four Year Olds**

Learning is accelerated if everything that is taught is within learners’ zone of proximal development. If this format of instruction is followed, “average” learners could learn things that are “developmentally” far beyond what children are assumed to be capable of learning. For example, we could teach young children all the component skills they need to work problems like 5/2K = 10 and word problems that are solved by multiplying values by their reciprocal (e.g., If 2/3 of the pennies weigh 12 pounds, how much do all the pennies weigh?).

In the 1960s, we taught math to children, including at-risk children, who started as four year olds and had math instruction 20 minutes a day. As kindergarteners, they could solve problems of the form 2/3A = 6. They also solved word problems that they expressed as equations and solved. (1/4 of a pie costs 5 cents. How much does the whole pie cost? 1/4P = 40).

Children understood that any fraction could be read as a division problem: in this case, 40 divided by 4. They also knew how to work simple problems that involve negative numbers and problems that had unknowns (7 - b = 2). They could rewrite equations to solve for different unknowns (A + B = C; therefore, A = C - B and B = C - A). They could solve area-of-rectangle problems, including those that gave the number of square units and the length of one pair of sides. The children figured out the length of the other pair of sides.
Their performance confirmed that if everything taught is sequenced so that it is within the children's zone of proximal development, they could learn math operations that most sixth graders haven't learned. Their performance also made a mockery of the notion of developmental readiness and norms. The children's entry skills indicated where the instruction should start. (Nearly all were unable to count to ten.) Their initial performance, however, does not predict what children will learn if the instruction builds sequences that scrupulously remain within the children's progressively changing zone of proximal development.

You be the judge. View this video of kindergarteners showing off their math skills: Kindergarteners Showing Off Their Math Skills